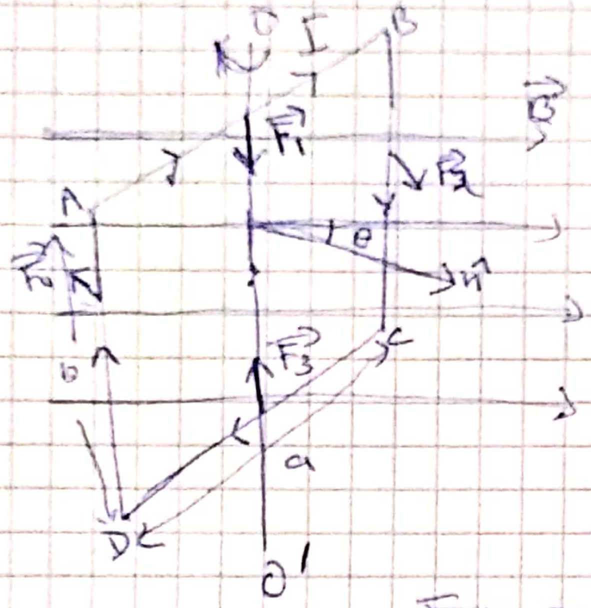
Torque on a Current loop in uniform mag field



Rectangular loop ABCD is free to rotate about OO' ⊥ to $\vec{B}$ (Through its COM)

$\hat{n}$ – normal to the plane of loop

Force on top side AB $= \vec{F_1} = I(\vec{l} \times \vec{B})$

$= I\,aB\sin(90° - \theta) = IaB\cos\theta$

$\vec{F_1}$ is in the downward direction.

Force on CD $= F_3 = IaB\cos\theta$ in upward dir.

$\vec{F_1} = -\vec{F_3}$ ∴ $\vec{F_1} + \vec{F_3} = 0$ (both have the same line of action)

Force on BC $= F_2 = I(\vec{l} \times \vec{B}) = IbB$ ($\vec{BC} \perp \vec{B}$)

$\vec{F_2}$ is along the outward normal to the plane of paper

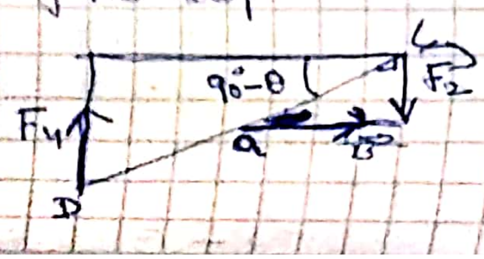
$\vec{F_4} = IbB$ along the inward " " " "

$\vec{F_2}$ & $\vec{F_4}$ are oppositely directed and do not have the same line of action. They constitute a couple.

Net Torque on the couple $= \tau = IbB \times a\cos(90-\theta)$

$= IBA\sin\theta$, $A = ab$ - Area of the loop

or $\vec{\tau} = I(\vec{A} \times \vec{B})$



Net torque tends to rotate the loop in the [illegible]

Eqn for $\vec{\tau}$ holds good for all plane loops of any shape having an area $\vec{A}$.

If a coil consists of N-turns,

$$\vec{\tau} = NI(\vec{A} \times \vec{B}) \quad \text{--- ①}$$

The coil is rotated by this torque until the eq. position reaches in which $\vec{A}$ is in the dir. of $\vec{B}$.

Torque on an electric dipole $\vec{\tau} = \vec{p} \times \vec{E}$ ②

From ① & ②, a current carrying loop behaves like a mag. dipole with dipole moment $\vec{P_m} = NI\vec{A}$
$= \vec{m}$

$$\left(\vec{\tau} = \vec{P_m} \times \vec{B} \right)$$

dir. of $\vec{P_m}$ or $\vec{m}$ is the dir of thumb if R-H. encircles the current loop or coil s.t. fingers point in the dir. of the current.

ⓐ If a mag. dipole is turned through an angle θ from the zero energy position ($\theta = \pi/2$) then work done

$$= U = \int_{90^\circ}^{\theta} \tau \, d\theta = P_m B \int_{90^\circ}^{\theta} \sin\theta \, d\theta = -P_m B \cos\theta$$

$$= -\vec{P_m} \cdot \vec{B}$$

This work done is stored as the pot. energy of the mag. dipole.

Magnetisation of Matter

Each e^- in an atom produces orbital & spin magnetic moments and can be assumed as a dipole.

$\vec{m}_e = -(e/2m_e)\vec{L}$ → orbital mag. moment.

$\vec{L}$ – Angular " of e^-

$\vec{m}_s = -(e/m_e)\vec{S}$ → spin mag. moment of e^-

$\vec{S}$ – Spin angular moment

Nucleus also contributes to the total mag. moment of atom but its about 10^{-3} times that of electronic moments.

In some materials, net mag. moment of material = 0 as these dipoles are randomly oriented due to thermal agitations.

When an external mag. field $\vec{B}$ is applied:

(i) Atoms having non zero mag. moment are aligned in the dir. of applied $\vec{B}$ due to torque τ. ⇒ Increase its mag. moment is along $\vec{B}$.

(ii) If atoms had a zero mag. moment, $\vec{B}$ distorts the e^- orbit and thus induces mag. moment.

Induced mag. moment is set up in the dir. opposite to $\vec{B}$. The mag field intensity inside the material will be less than that outside the body and the material is called diamagnetic material

e.g. Cu, Bi, Zn, Ag, Pb, Hg etc.

In Paramagnetic substances, atoms have intrinsic mag. moments (unpaired e^-s) when external mag. field is applied both (i) & (ii) takes place and the resultant mag. moment is always in the direction of mag. field B & (i) dominates over (ii).

$\therefore$ Material body can be considered as a volume distribution of magnetic dipoles ~~if~~ acted upon by an external mag. field.

If $\vec{m} \rightarrow$ av. mag. dipole moment per atom or molecule

N — No. of such effective dipoles per unit vol.

Then $\vec{M} = N\vec{m}$ Magnetisation vector = Net mag. dipole polarisation per unit volume.

If m of all atoms are not parallel to each other, then

$$\vec{M} = \lim_{\Delta v \to 0} \frac{1}{\Delta v} \sum_{i=1}^{N} \vec{m}_i \qquad \therefore \vec{M} - \text{vector point fn.}$$

$\vec{M} = 0$ if the material is unmagnetised.

When $\vec{B}$ is applied orientation of m is changed.

degree of magnetisation depends on the applied field as well as the material.

In vacuum, $\vec{B}$ inside a solenoid $= \mu_0 (NI/l)$

$$= \mu_0 J_s^*$$

Where $J_s^* = NI/l$ — Solenoid current density.

units are amp/m.

If inside of a solenoid is filled with a material then $\vec{B}$ inside the solenoid $= \vec{B}$ due to J^*_{free} + $\vec{B}$ due to magnetisation of matter.

$\vec{B}$ due to magnetisation of matter results from the rotating & circulating e's. Thus constitute a current known as atomic current.

External current corresponding to e^- transport is called free current.

Atomic current is defined in terms of an effective solenoid current density $\vec{J}^*_{mag}$ that describes the mag. effect of the matter.

Consider a cylindrical rod of length 1 meter having uniform magnetisation parallel to its axis
⇒ uniform density of circulating currents everywhere inside the material.

In a small slice of cylinder, may be considered as made up of large no. of cells of very small area on the surface of every cell, atomic currents ~~in each~~ in adjoining cells cancel each other. Only circulating currents around the outer surface of the slice will be non zero.

∴ There is net circulating current only on the surface of the rod.

⇒ uniformly magnetised rod is equivalent to a long solenoid carrying an electric current known as Amperian current.

For a cylinder of length l & area of X-section S, mag. moment around the periphery = sum of mag. moment of all the cells.

m = circulating current × area of the loop

$= l \times$ effective solenoid current density $J^*_{mag} \cdot S$

$= l\, J^{s}_{mag}\, S = M \cdot$ volume of cylinder

$= M l S$

or $\boxed{M = J^{s}_{mag}}$ ⟹ surface current flowing per unit length at the surface of the magnetised vol. = Magnetisation M.

Also called surface mag. current density.

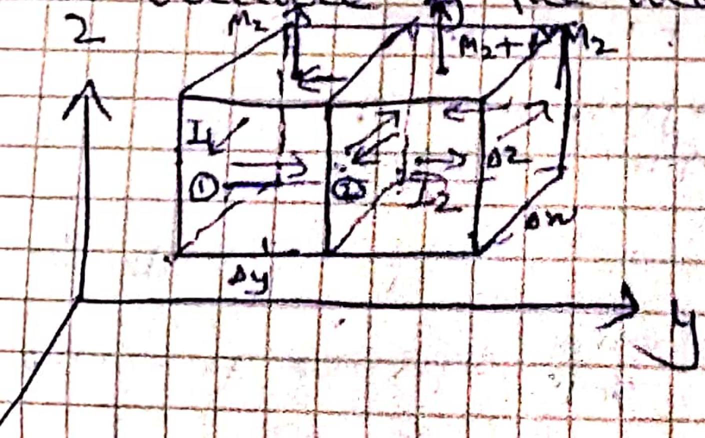
In vector form, $\vec{J}^{s}_{mag} = \vec{M} \times \hat{n}$

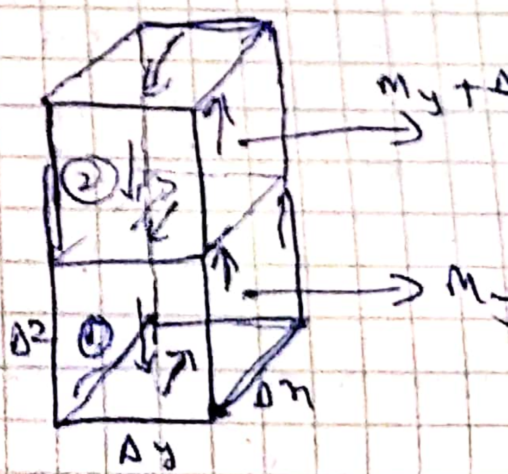
$\hat{n}$ – outward normal to the surface.

Non-Uniform Magnetisation:

$\vec{M}$ – varies from point to point & it's a vector point fn.

In non-uniform magnetisation, the cancellation will not be complete & there will be a net current in volume of the material.





M_z – comp of $\vec{M}$ along z-axis on element 1

$M_z + \Delta M_z$ comp of $\vec{M}$ " " " " " 2

$$\Delta M_z = (\partial M_z/\partial y)\,\Delta y$$

I_1 & I_2 circulating currents in elements ① & ②

$\therefore\ M_z \Delta x \Delta y \Delta z = I_1 \Delta x \Delta y$ & $(M_z + \Delta M_z)\,\Delta x \Delta y \Delta z = I_2 \Delta x \Delta y$

$\Rightarrow\ I_2 - I_1 = \Delta M_z \Delta z = \left(\frac{\partial M_z}{\partial y}\right)\Delta y \Delta z$ – Net current on the common boundary of two elements due to circulating currents I_1 & I_2

It is along x-dir.

$\therefore\ J_{x,1} = I_x/\Delta y \Delta z$ = x-comp of current density

$$J_{x,1} = \left(\frac{\partial M_z}{\partial y}\right)$$

Similarly M_y – comp of $\vec{M}$ along y-dir. on element 1

$M_y + \Delta M_y \to$ " " " " " " ②

$$\Delta M_y = (\partial M_y/\partial z)\,\Delta z$$

Net current in the common boundary of two elements due to circulating current I_1' & I_2' which are in the x-z plane is given by

$$I_1' - I_2' = \Delta M_y \Delta y = -(\partial M_y/\partial z)\,\Delta y \Delta z$$

[$M_y \Delta x \Delta y \Delta z = I_1' \Delta x \Delta z$; $(M_y + \Delta M_y)\,\Delta x \Delta y \Delta z = I_2' \Delta x \Delta z$]

$\therefore$ x-comp. of current density

$$J_{x,2} = -\partial M_y/\partial z$$

Since the only surfaces of the elements which contribute to the current density in the x-dir. are x-y & x-z surfaces.

$\therefore$ Net current density in x-dir. is

$$J_{m,x} = J_{m,1} + J_{m,2} = \left(\frac{\partial M_z}{\partial y} - \frac{\partial M_y}{\partial z}\right) = (\vec{\nabla} \times \vec{M})_x$$

Similar expressions may be obtained for y- & z- compo. of $\vec{J}_m$

i.e. $\vec{J}_{m,y} = (\vec{\nabla} \times \vec{M})_y$ & $J_{m,z} = (\vec{\nabla} \times M)_z$

$$\boxed{\vec{J}_{m,v} = \vec{\nabla} \times \vec{M}}$$

→ Net volume density of atomic current due to incomplete cancellation of adjacent current loops in the interior of magnetised body.

Thus one can calculate $\vec{B}$ by replacing the magnetic material by its equivalent currents $\vec{J}_{m,s}$ & $\vec{J}_{m,v}$.

Magnetic Intensity $\vec{H}$

For steady currents & non-mag. materials

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

Value of $\vec{B}$ changes in the presence of mag. field material due to magnetisation.

Effect of magnetisation is equivalent to the mag. field produced due to equivalent solenoid currents J_m known as bound current (due to surface

& volume magnetisation currents associated with the mol. or atomic magnetic moments.)

$\therefore$ Total mag. field $\vec{B}$ is given by

$$\vec{\nabla} \times \vec{B} = \mu_0 (\vec{J}_f + \vec{J}_m)$$

Since $J_{m,v} = \vec{\nabla} \times \vec{M}$

$$\therefore \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}_f + \mu_0 \vec{\nabla} \times \vec{M}$$

or, $\vec{\nabla} \times (\vec{B} - \mu_0 \vec{M}) = \mu_0 \vec{J}_f$

$\vec{\nabla} \times \vec{H} = \vec{J}_f$ where $\boxed{\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}}$

As $\vec{\nabla} \cdot \vec{B} = 0$, $\Rightarrow \vec{\nabla} \cdot (\vec{H} + \vec{M}) = 0$

or $\boxed{\vec{\nabla} \cdot \vec{H} = - \vec{\nabla} \cdot \vec{M}}$

$\therefore \vec{\nabla} \cdot \vec{H} \neq 0$ if $\vec{\nabla} \cdot \vec{M} \neq 0$

and, $\oint_S (\vec{\nabla} \times \vec{H}) \cdot \vec{ds} = \int_S \vec{J}_f \cdot \vec{ds} = I_f$ — Free Currents.

or $\boxed{\oint_C \vec{H} \cdot \vec{dl} = I_f}$ — Amp. Circuital law

C – Curve bounding surface S.
I_f – free currents flowing through C.

$\oint \vec{H} \cdot \vec{dl}$ = Magnetomotance

as $\oint \vec{E} \cdot \vec{dl}$ = electromotive force in Electric fields.

Units of H are Amp./met.

If there is no free current then $\oint_C H \cdot dl = 0$

Lines of $\vec{H}$ are as if the magnetic poles really were the source of magnetisation, instead of electric current.

$\therefore$ ① $\vec{H} = \vec{B}/\mu_0 - \vec{M}$ in mag. material

$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$ in dielectric ".

② $\oint \vec{H} \cdot \vec{dl} = I_f$ free currents does not include magnetisation currents.

$\oint_S \vec{D} \cdot \vec{ds} = q_f$ — free charges does not include polarisation charges

↓

Gauss's Theorem.

Amp. Circ. law

$\vec{B} \longrightarrow$ Magnetic Induction vector.

Magnetic Susceptibility & Permeability

When $\vec{H}$ is applied, mag. mom. developed in mag. substances. Magnetisation $\propto$ Applied field.

or $M \propto H$ [except at fairly low temp and when in very strong mag. fields.]

or $M = \chi_m H$

χ_m — Const of prop. — Magnetic Susceptibility, or Volume ")

→ Which is a measure of how susceptible the matter is to polarization by an applied mag. field. Which is a characteristic of the material.

If $\vec{M}_{mass}$ = mag. moment of a unit mass of material

then $\vec{M}_{mass} = \chi_{m,mass}\vec{H}$ or $\boxed{\chi_{m,mass} = \chi_m/\rho}$

If mag. moment of one molecule is $\vec{M}_{mol}$ then

$\vec{M}_{mol} = \chi_{m,mol}\vec{H}$ or $\boxed{\chi_{m,mol} = \chi_m A/\rho}$

ρ – density & A – molecular weight of the material

& $\chi_{m,mass}$ → ~~molar~~ mass susceptibility

$\chi_{m,mol}$ → Molar "

χ_m → +ve for Paramagnetic substances

→ −ve for diamag. "

and is always less than 1.

and $\boxed{\chi_m = C/T}$ → χ_m decrease with temp for paramag. substances

C – const — Curie const

and χ_m is independent of Temp. for diamag. substances

Now $\vec{B} = \mu_0(\vec{H}+\vec{M}) = \mu_0(1+\chi_m)H = \mu H$

$\boxed{\mu = \mu_0(1+\chi_m)}$ → mag. permeability of material

$\frac{\mu}{\mu_0} = K_m$ → Relative " of material

$K_m = 1$ for vacuum, μ_0 → Permeability of vacuum

Km is slightly ~~less~~ greater than 1 ~~for~~ for paramag. substance

" " " less " 1 " diamagnetic "

For diamag. $\vec{M}$ & $\vec{H}$ are antiparallel

" Paramag. " " " " parallel.

Permeability is a measure of the degree of penetration of mag. field through the substance.

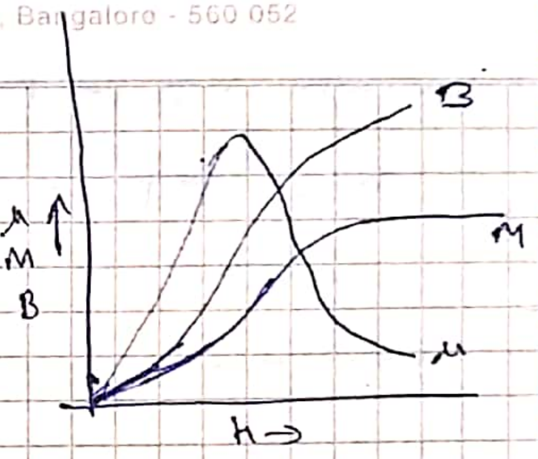
Ferromagnetism: Like Paramagnets, these substances have a high degree of magnetization.

* Strongly magnetized by even a weak mag. field.
* Relative Permeability is very large ~100 to 1500.
* Susceptibility ~~is~~ the ~~is~~ very large.
* $M \propto H$ for small values of H, increases rapidly for larger values and attains a const value for very large H.

* χ_m → const for small H
→ increases for larger values of H
→ decreases for very large values of H.

* B - varies as M with H except that B $\not\to$ const at very values of H.

* M → varies as χ except at very high H where M decreases slowly in comparison to χ.



* χ decreases with temp. Above a certain temp, Curie temp, ferromagnetic substances become paramagnetic. $[\chi = C/T-T_c]$

T_c – 1000 for iron, 770 for steel, 360 for Nickel & 1100 °C for cobalt.

(Magnetization is due to mag. moments of e^- spin)

In Ferromagnetic substances the effective field acting on each spin magnetic dipole is the vector sum of the applied field + strong interaction field arising from all the neighbouring dipoles. This interaction field is so strong that thermal vibrations even at room temp. cannot destroy the alignment. This interaction is known as spin - spin or exchange interaction.

× Magnetization exists even in the absence of external magnetic field, - called Spontaneous Magnetization.

* Above Curie temp T_c, spontaneous magnetization vanishes and the ferromagnetic material behaves as ordinary paramagnetic material.

Susceptibility $\chi = \frac{M}{H_0} = C/T-T_c$ — Curie-Weiss Law.

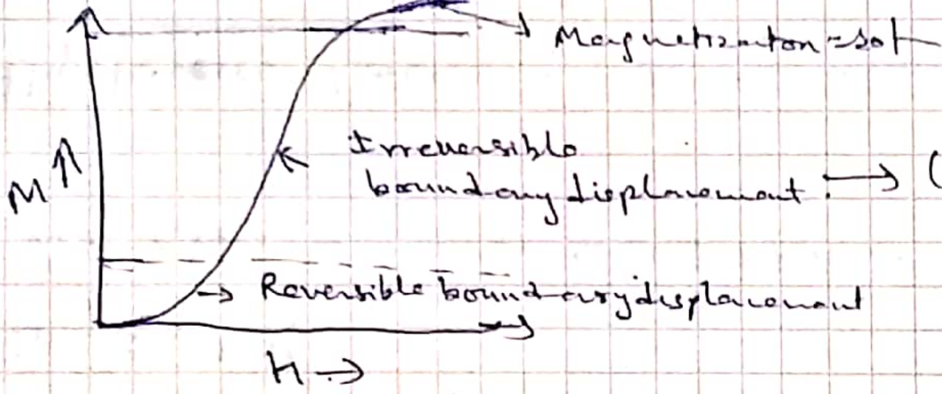
For $T < T_c$, the law is not applicable & a finite value of magnetization exists even when applied field $H_0 = 0$. This mag. is called spontaneous mag.

Ferromagnetic domains: A domain is a region composed of no. of elementary magnets, each pointing with its axis in the same dir. even in the absence of any external field. Size of the domain is 10^{-6} cm^3 to 10^{-2} cm^3. In an unmagnetized ferromag. specimen the domains are oriented at random so their resultant mag. mom. is zero.

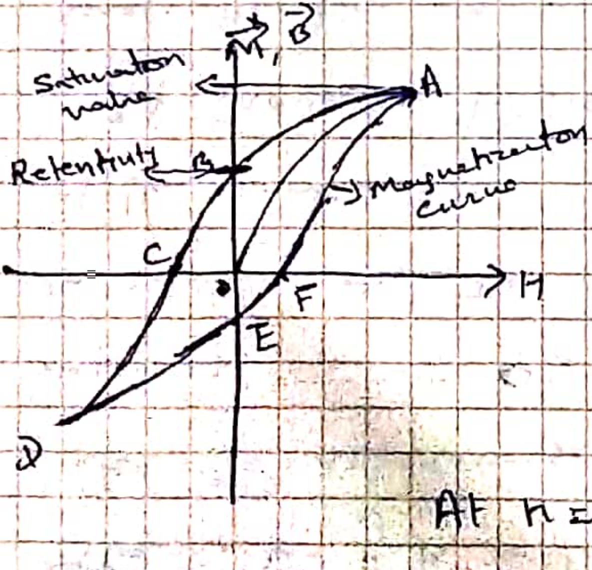
When the specimen is placed in an external field, magnetization increases in 2-ways:

(i) Increase in the vol. of the domains which are favourably oriented w.r.t. the field at the expense of unfavourably oriented domains. → occur in weak fields

(ii) By Rotation of the domain magnetization towards the field dir. → occur in strong fields.



Hysteresis : In Ferromagnets, $\vec{M}$ is not directly proportional to $\vec{H}$. If an unmagnetised F.M. material is put in a mag. field $\vec{H}$ and



As $\vec{H}$ increases from 0 gradually, M increase non-linearly and reaches a max. value at A. At this point all the dipoles are aligned in the dir. of $\vec{H}$.

As $\vec{H}$ → decreases, M decrease along AB not along OA (Mag. curve)

At H = 0, M ≠ 0, the value of M is called remanance, remanent magnetization or retentivity

$\boxed{\vec{B} = \mu_0(H+M)}$

The value of $\vec{B} = \mu_0 M$ is called the residual induction.
As $\vec{H}$ is reversed, $\vec{M}$ decreases and finally becomes zero at C, OC → reversed mag. field needed to demagnetize the specimen called the coercivity of the material.

If reverse $\vec{H}$ is further increased, reverse magnetization is ~~increased~~ set up which quickly reaches the saturation value at D.

If $\vec{H}$ is now increased (or reverse $\vec{H}$ decreased) the saturation value is achieved along the curve D, E, FA.

. The closed loop ABCDEFA → Hysteresis loop.

Hysteresis Loss : In an unmagnetised specimen, the net magnetization is zero as the magnetization of different domains is ~~in diff~~ randomly oriented.
In an external mag field, the domains are rotated so that $\vec{M}$ is in the dir of $\vec{H}$.
∴ Work is done & energy is taken by the specimen.
When $\vec{H}$ is removed, $\vec{M} \neq 0$, domain do not regain their original positions ⇒ energy is not

recovered during demagnetization & there is always a definite loss of energy of each hysteresis loop.
This energy loss is known as Hysteresis loss.
$= \mu_0$ Area of M-H loop or = Area of B-H curve.